Stage 9

Mathematics

- understand and use the concepts and vocabulary of inequalities
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- represent the solution set to an inequality on a number line
- use compound units such as density and pressure
- calculate arc lengths, angles and areas of sectors of circles
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations and know the underlying assumptions
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- translate simple situations or procedures into algebraic expressions or formulae
- construct plans and elevations of 3D shapes
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$
- find approximate solutions to simultaneous equations using a graph
- identify and apply circle definitions and properties, including: tangent, arc, sector and segment
- enumerate sets and combinations of sets systematically, using tree diagrams
- understand and use the concepts and vocabulary of identities
- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- apply and interpret limits of accuracy
- plot and graph (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to simple kinematic problems involving distance, speed and acceleration
- interpret (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to simple kinematic problems involving distance, speed and acceleration
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
- draw estimated lines of best fit; make predictions
- use inequality notation to specify simple error intervals due to truncation or rounding
- recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = 1/x$ with $x \neq 0$
- use these to construct figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
- give construct to
- interpret and construct tables, charts and diagrams, including tables and line graphs for time series data and know their appropriate use
- calculate with standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- recognise, sketch and interpret graphs of quadratic functions
- use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures
- calculate exactly with multiples of π
- find the equation of the line through two given points, or through one point with a given gradient
- change freely between compound units (e.g. density, pressure) in numerical and algebraic contexts
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- calculate with roots, and with integer indices
- use the form $y = mx + c$ to identify parallel lines
- solve linear inequalities in one variable
- recognise and use Fibonacci type sequences, quadratic sequences
- interpret the gradient of a straight line graph as a rate of change;
- *calculate surface area of right prisms (including cylinders)*

1 **a)** Find the value of ■:

i) $2^{■} = 32$

$■ = $ ....

ii) $\sqrt[■]{64} = 4$

$■ = $ ....

iii) $8^{■} = 1$

$■ = $ ....

iv) $\sqrt[■]{625} = 5$

$■ = $ ....

v) $(-2)^{■} = -8$

$■ = $ ....

vi) $\sqrt[3]{-27} = ■$

$■ = $ ....

(NSC1, 6 marks)

2 The area of an annulus can be calculated using the formula:

$$A = \pi(R^2 - r^2)$$

Find the area of an annulus with *R* = 8 cm and *r* = 3 cm, leaving the answer in terms of π.

....

(NSC2, 2 marks)

3 **a)** Calculate, leaving your answer in standard form:

i) $(1.6 \times 10^4) \times (2 \times 10^5)$

....

ii) $(2.4 \times 10^{16}) \div (2 \times 10^8)$

....

iii) $(3 \times 10^{-4}) \times (6 \times 10^{-5})$

....

iv) $(4 \times 10^{-9}) \div (8 \times 10^{-5})$

....

(NSC3, 12 marks)

4 **a)** Use inequalities notation to write down the error interval:

i) $x = 3.2$ rounded to one decimal place

.... $\leq x <$ ....

ii) $x = 1.64$ rounded to two decimal places

....

iii) $y = 200$ rounded to nearest 100

....

iv) $t = 20$ rounded to one significant figure

....

v) $s = 3.2$ **truncated** to one decimal place

....

vi) $p = 1.64$ **truncated** to two decimal places

....

(NMA1, 12 marks)

5 **a)** Usain Bolt ran a 100 metre race in 9.58 seconds (rounded to the nearest hundredth of a second).

What is the fastest time he could have run the race (to three decimal places)?

....

b) Planks of wood are cut to a length of 80 cm (to the nearest cm).

What is the shortest possible length of wood?

....

(NMA2, 2 marks)

6 **a)** Look at the statements below.

$6m + 4n$	$4x - 11 = 9$	$2(a + b) = 2a + 2b$	$y = 6x + 2$
$n + n + n = 3n$	$2p + 1 = 5p - 7$	$A = l \times w$	$3p - 4q$

i) Tick the statements that are identities.

ii) Put a ring around the equations that are formulae.

(ANVM1, 4 marks)

7 **a)** Expand and simplify:

i) $(x + 2)(x + 4)$ **ii)** $(x + 2)(x - 4)$

………………………   …………………………

iii) $(a - b)(a + 3b)$ **iv)** $(x - y)(x - 2y)$

………………………   …………………………

v) $(x + 2)^2$

vi) $(x – 3)^2$

....

....

b) Factorise:

i) $x^2 + 7x + 12$

ii) $x^2 + 11x + 10$

....

....

iii) $x^2 + 4x – 12$

iv) $p^2 – p – 20$

....

....

v) $x^2 – 16$

vi) $y^2 – 25$

....

....

(ANVM2, 24 marks)

8 **a)** Look at the words here:

<div style="text-align:center">

| **Equation** | **Identity** |

</div>

Choose a word that best describes each algebraic statement.

Statement   **Description**

i) $10 = 4x - 3$: ....

ii) $4(x - 2) = 20$: ....

iii) $3(a + 5) = 3a + 15$: ....

iv) $4(p + 5) = 4p + 20$: ....

v) $(x + 2)(x + 3) = 7$: ....

vi) $n(n + 3) = n^2 + 3n$: ....

(ANVM3, 6 marks)

9 **a)** Show that:

$$4(x + 2) - 3(x + 6) = x - 10$$

b) Prove that:

$$(x + 3)^2 \neq x^2 + 9$$

c) Prove that this set of instructions will always give an answer of 6

- *Pick any number*
- *Double it*
- *Add 12 to the running total*
- *Halve the running total*
- *Subtract the original number from the running total*

(ANVM4, 6 marks)

10 **a)** Use inequality notation to represent the following:

i) x is greater than 5

…………………………

ii) x is less than or equal to -2

…………………………

iii) x is greater than 6 but less than 8.5

…………………………

iv) x is less than 6 but greater than 4.25

…………………………

v)

…………………………
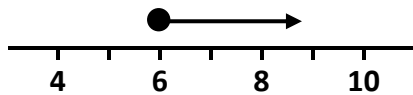
vi)

…………………………
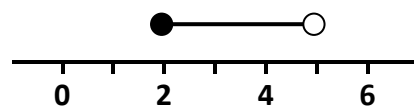
(ANVM5, 12 marks)

11   Look at the graphs here
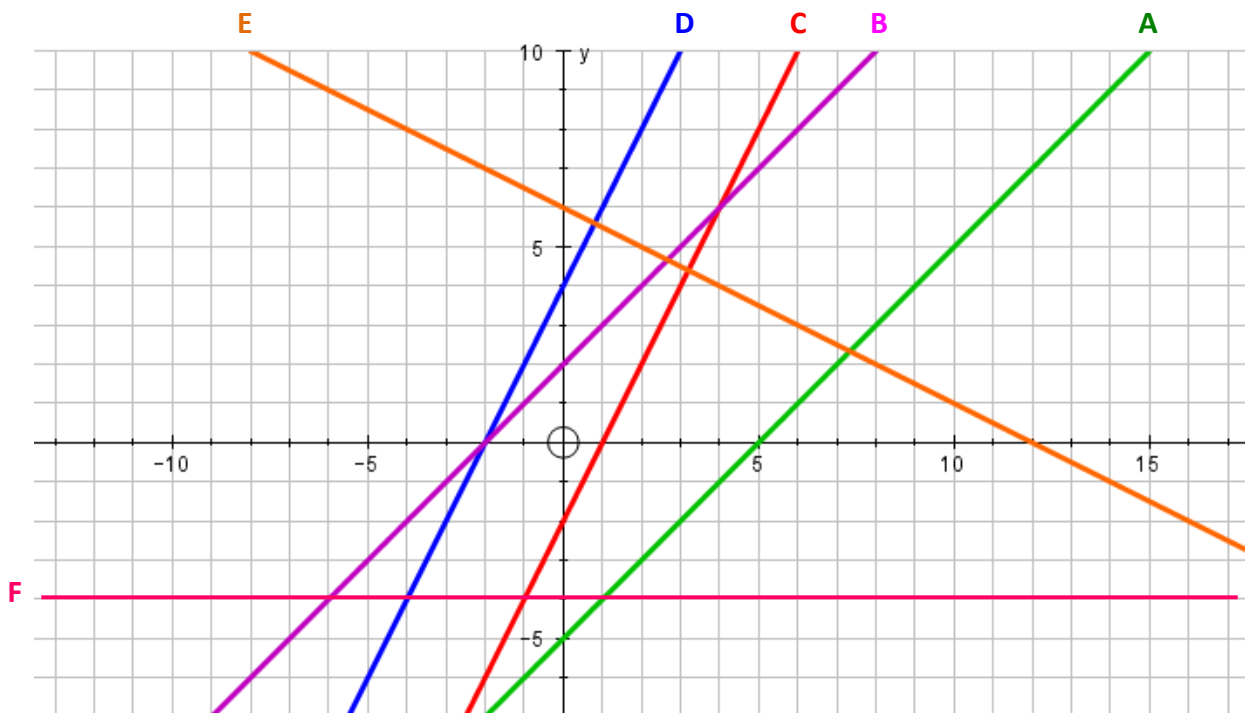
Using the graph, find the equations of the six lines

A	y = ...

D	

B	

E	

C	

F	

(AG1, 12 marks)

12   Which of the following graphs are parallel? Tick the ones that are.

$y = 3x - 5$ $y = 5x - 3$ $y + 3x = 5$

$3x - 2 = y$ $2y - 6x = 4$ $y = 10 + 3x$

(AG2, 3 marks)

13 a) Find the equation of line passing through the points (-1, 2) and (3, 10).

....

b) Find the equation of line with gradient 5 passing through the points (2, -3).

....

(AG3, 5 marks)

14 a) Look at the graphs here
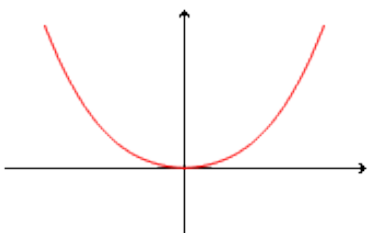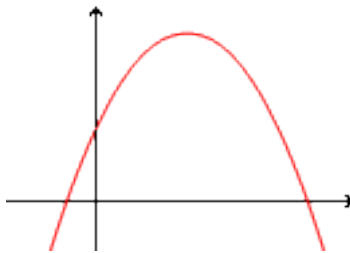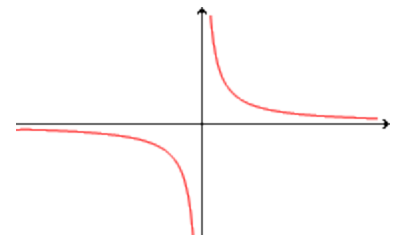
| **Graph 1** | **Graph 2** | **Graph 3** |

| **Graph 4** | **Graph 5** | **Graph 6** | **Graph 7** |
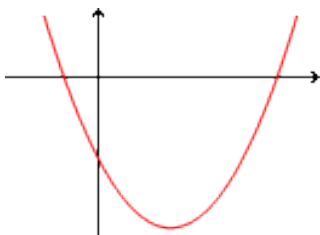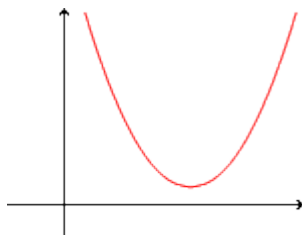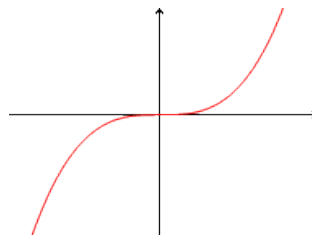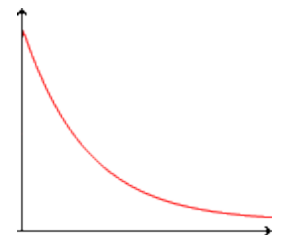
i) Which graphs are quadratic functions?   ....

ii) Which graphs are cubic functions?   ....

iii) Which graphs are reciprocal functions?   ....

b)   Sketch the graphs of

i) $y = x^2 + 3$
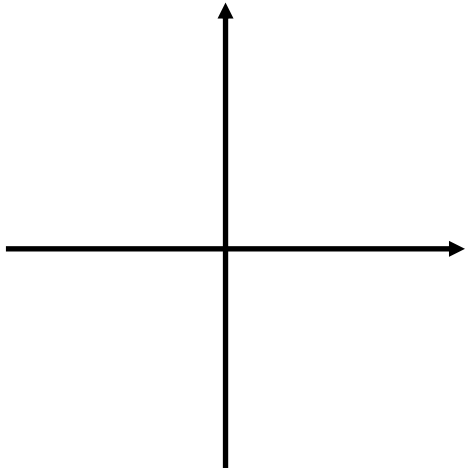
ii) $y = x^3$
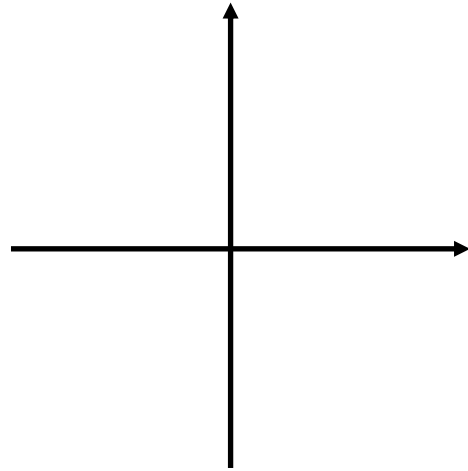
iii) $y = \dfrac{1}{x}$
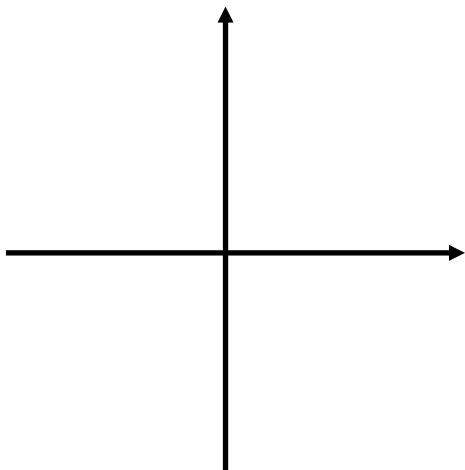
iv) $y = x^2 - 4$
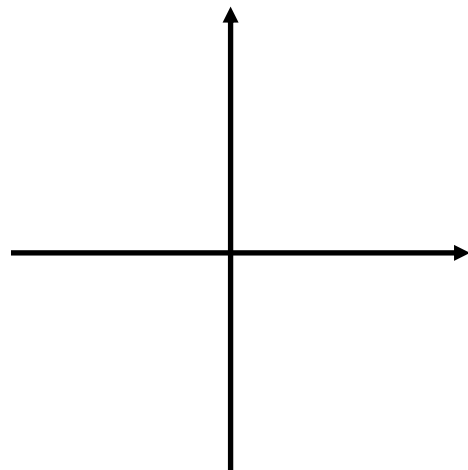
(AG4, 4 marks)
(AG5, 4 marks)

15 a) Plot the graph of $y = \dfrac{2}{x}$

x	-4	-3	-2	-1	0	1	2	3	4
y									
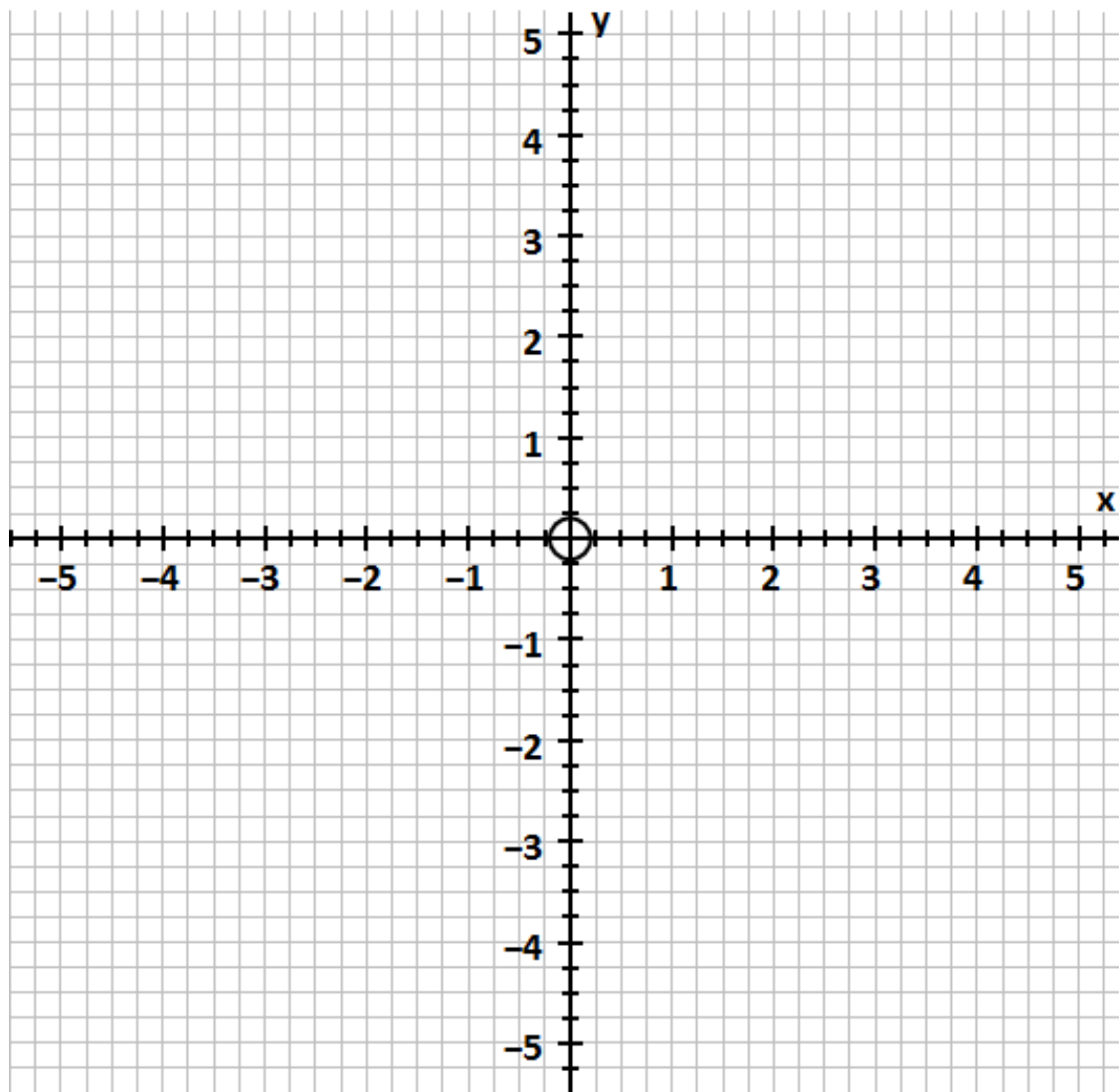
b) Pete is filming some aerial video using a drone. The graph shows information about the flight.

i) How far does the drone fly?

…………………… m

ii) Describe what is happening between 50 and 80 seconds?

iii) What is the speed of the drone 90 seconds into the flight?

…………………… m/s

iv) What is the speed of the drone on the return leg of the flight?

…………………… m/s

v) Describe what is happening between 0 and 50 seconds

(AG6, 12 marks)

16   Solve the simultaneous equations:

a) $3x + y = 22$
$x + y = 12$

b) $y = x + 9$
$2x + 2y = 30$

$x = $ ....

$y = $ ....

$x = $ ....

$y = $ ....

c) $3b + a = 31$
$b - a = 5$

d) $2b + 3a = 40$
$b = 2a + 13$

$x = $ ....

$y = $ ....

$x = $ ....

$y = $ ....

(ASE1, 12 marks)

17　　Use the graph to find an approximate
　　　　solution for the simultaneous equations:

　　　　$y = 4x - 3$
　　　　$2y + x + 4 = 0$

　　　　　　　　　$x = $ ....

　　　　　　　　　$y = $ ....
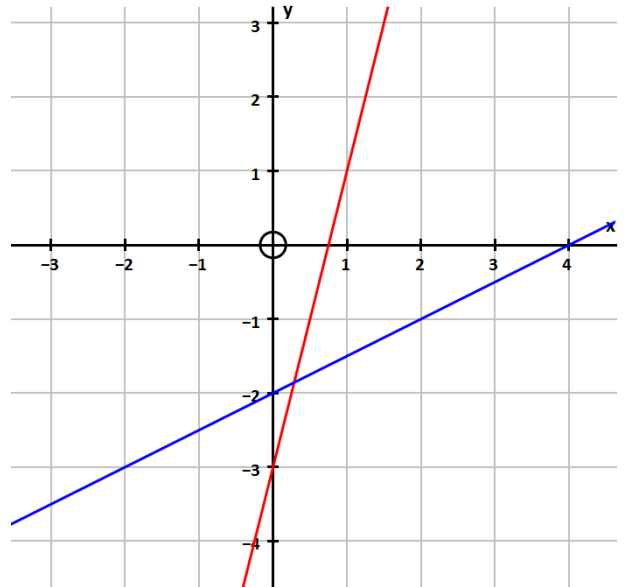
(ASE2, 2 marks)

18　　A rectangle has length 3 centimetres longer than the width.

　a)　Write an expression for the length of the rectangle

　　　　　　　　$(................)$ cm

　　　　　　　　k cm
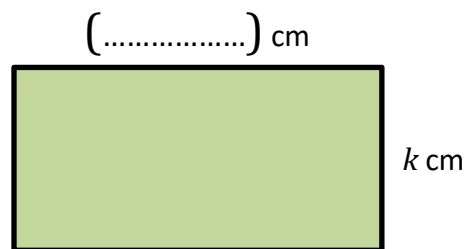
　b)　Write a formula for the perimeter of the rectangle

　　　　　　　　　　　　　　$P = $ ....

　c)　Write a formula for the area of the rectangle

　　　　　　　　　　　　　　............................

(ASE3, 3 marks)

19 a) A rectangle has width 2 centimetres less than the length.

x cm

$(x-2)$ cm
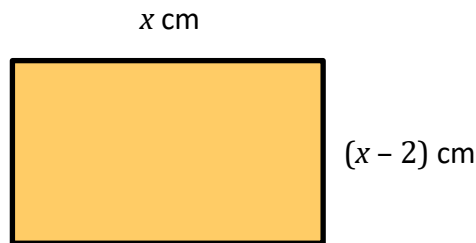
The perimeter of the rectangle is 10 cm.

i) Write an equation for the perimeter of the rectangle.

....

ii) Solve the equation to find the value of x.

$x =$ ....

b) Three cups of tea and two cups of coffee cost £6.60.

Five cups of tea and one cup of coffee cost £7.50.

i) Write a pair of simultaneous equations to describe this situation

....

....

ii) Solve the simultaneous equations to find the cost of a cup of tea and the cost of a cup of coffee

A cup of tea costs ....

A cup of coffee costs ....

(ASE4, 8 marks)

20   For each inequality:

     i)   Solve to find the range of values for x

     ii)   Represent the solution on a number line

     a) $2x + 5 < 23$ **b)** $4x - 6 \geq 42$

....

     c) $45 > 5(2x + 3)$ **d)** $6 + 2x \leq x - 3$

....

(ASE5, 10 marks)
(ASE6, 4 marks)

21 a)   Describe the following sequences using the terms:

Fibonacci-type	**Quadratic**

i)   1, 3, 4, 7, 11   : …………………………………………………….

ii)   1, 4, 9, 16, 25   : …………………………………………………….

iii)   1, 1, 2, 3, 5   : …………………………………………………….

iv)   2, 5, 9, 14, 20   : …………………………………………………….

b)   Find the 10th term of the sequences:

i)   4, 8, 13, 19, 26, …

…………………….

ii)   2, 3, 5, 8, 13, …

…………………….
(AS1, 7 marks)

22   Are these statements true (T) or false (F)?

a)   100 kg/m^2 is equivalent to 1 kg/cm^2 ………

b)   1 kg/m^3 is equivalent to 1000g/m^3 ………

c)   £36 000 annual salary is equivalent to £3000 per month   ………

d)   100 mph is equivalent to 10 miles/minute   ………

(RPRC1, 4 marks)

23 a)   The density of gold is 19.3 g/cm^3.

The volume of a gold bar is approximately 650 cm^3.

What is the mass of the gold bar?

....

b) John is wearing trainers. He balances on one leg.

If John weighs 65 kg and the sole of the trainer has an area of 120 cm^2, calculate the pressure exerted on the ground through the trainer in g/cm^2.

.... g/cm^2

(RPRC2, 4 marks)

24   The graph shows a journey
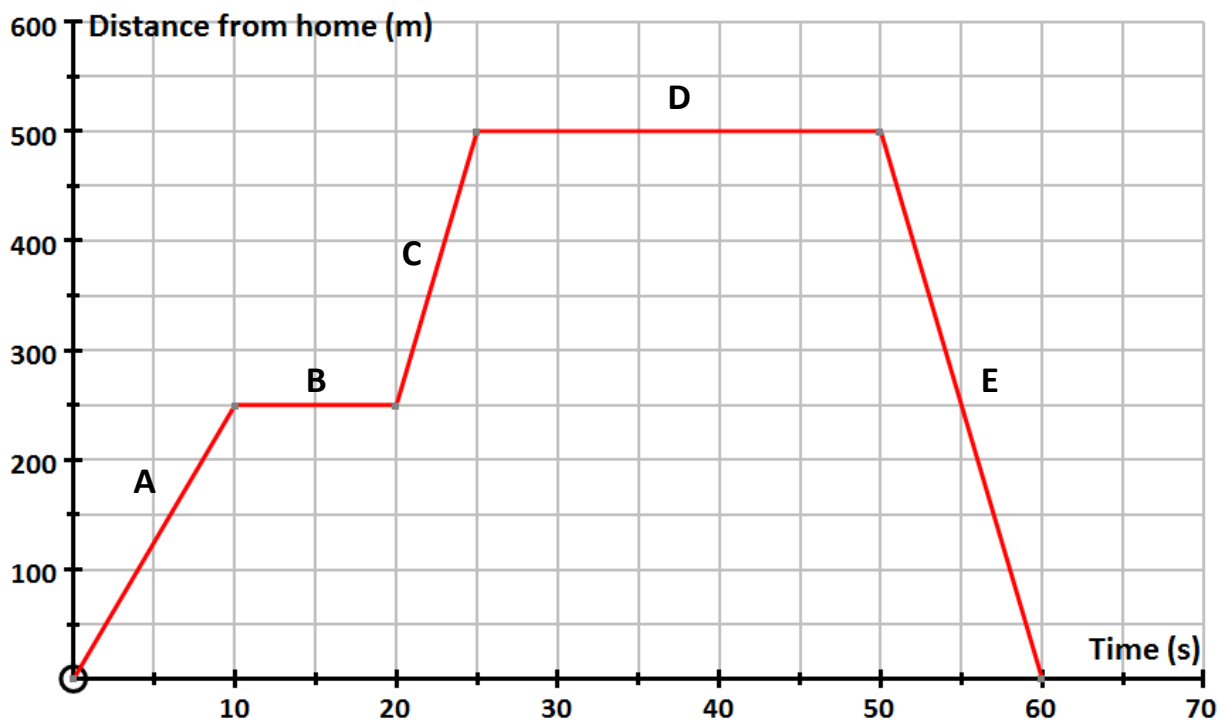
Calculate the velocity in each of these sections:

A	.... m/s

B	.... m/s

C	.... m/s

E	.... m/s

(RPRC3, 4 marks)

25 **a)** The amount Phil is paid is directly proportional to the number of hours he works.

Phil earns £47 for 6 hours' work.

How much will Phil earn for 9 hours' work?

£ ....

b) The number of sandwiches available is inversely proportional to the number of guests.

When there are 48 guests there are 5 sandwiches each.

How many sandwiches are available when there are 40 guests?

....
(RPRC4, 4 marks)

26 a)   Construct the perpendicular bisector of the line segment.
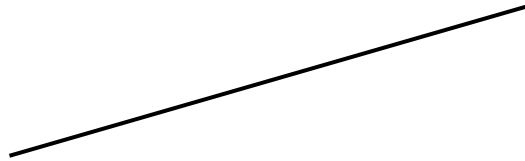
b)   Bisect the angle
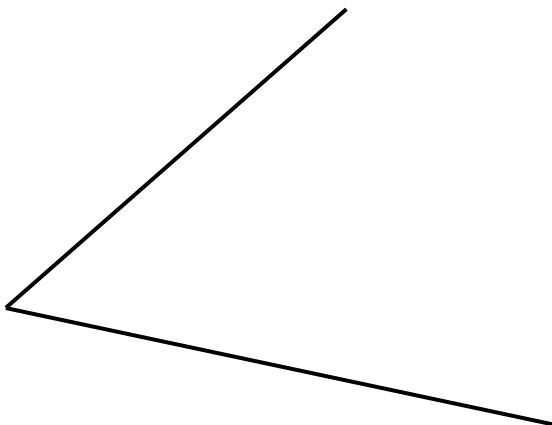
c)   Construct the perpendicular to the line at point A.

A
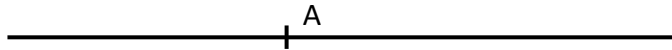
(GMPC1, 6 marks)

27   George is planting a lawn. The grass must be must be at least 3 metres from the garden wall and 2 metres from the tree (X).

Using a scale of 1 centimetre to 1 metre, shade the area where George should plant the lawn.

Garden wall

×
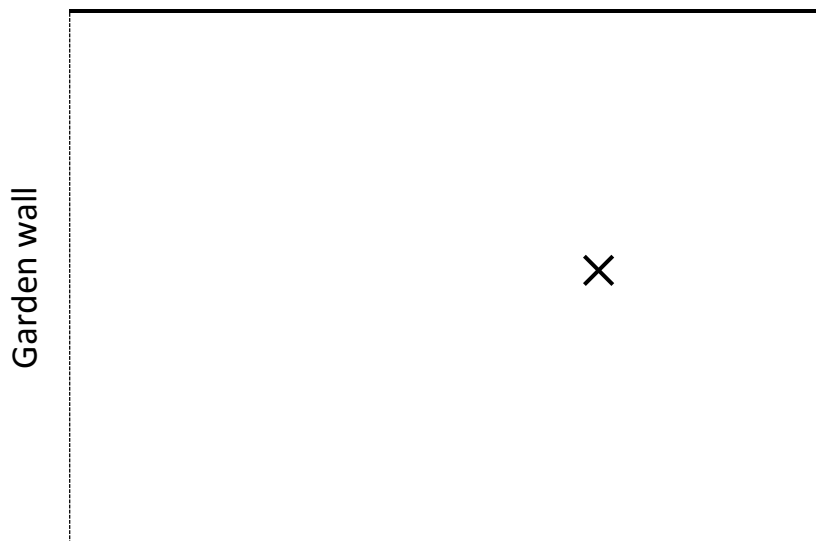
Scale 1 cm to 1 m

(GMPC2, 3 marks)

28   PQRS is a parallelogram.

Prove that triangle PQS is congruent to triangle QRS.
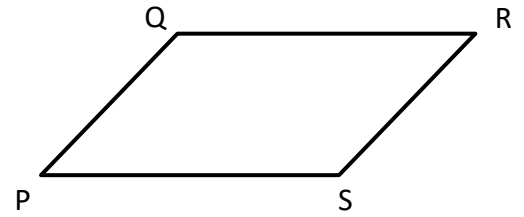
(GMPC3, 3 marks)

29   List all the basic congruence criteria for triangles.

(GMPC4, 4 marks)

30   Using the circle, identify and label:

a)   tangent

b)   arc

c)   sector

d)   segment
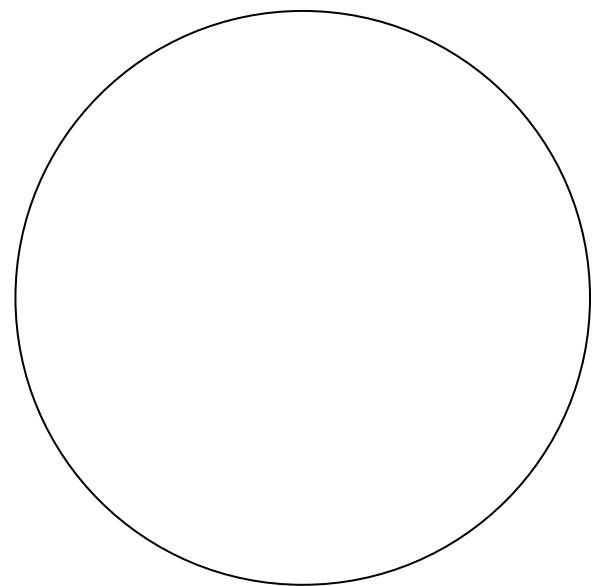
(GMPC5, 4 marks)

31   This solid is made from 6 cubes.

Construct and label:

a)   the plan

b)   the front elevation

c)   the left elevation

LEFT

FRONT

(GMPC6, 3 marks)

32   Find the following solutions to three significant figures:

a)   Find the **area** of the sector

8 cm
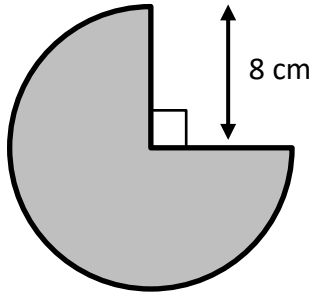
.... cm^2

b)   Find the **area** of the shaded section

6 cm
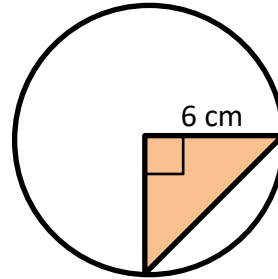
.... cm^2

c)   Find the **area** of the sector

60°

8 cm
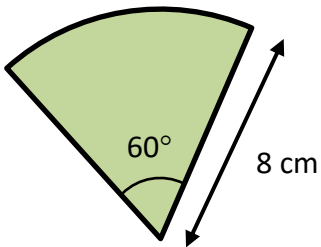
.... cm^2

d)   Find the **perimeter** of the sector

240°

4 cm
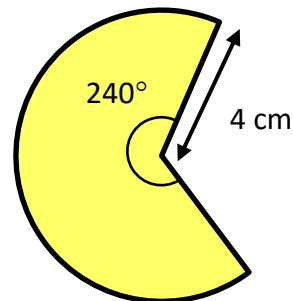
.... cm

(GMMC1, 11 marks)

33 a) Calculate the surface area of this prism:

8 cm

10 cm

10 cm

6 cm
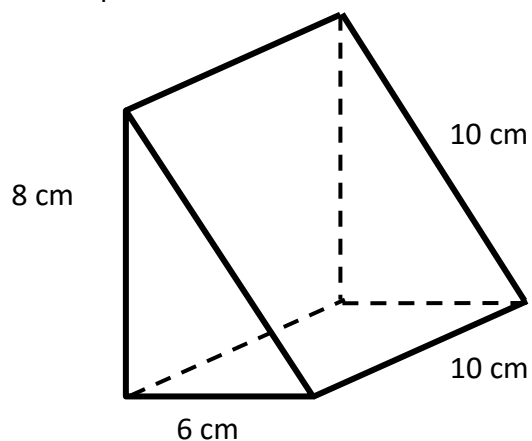
.... cm^2

b) Calculate the surface area of a cylinder with base radius of 5 cm and height 10 cm.

.... cm^2

(GMMC2, 6 marks)

34   Every pair of shapes is similar. Calculate the missing sides:

a)

?

20

1.2

4.8

? = ……………………

b)

9

6

9

x

x = ……………………

c)

3

a

2

0.6

a = …………………

d)

p

2.4

7.2

9.3
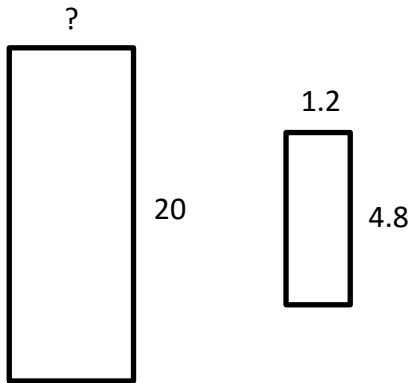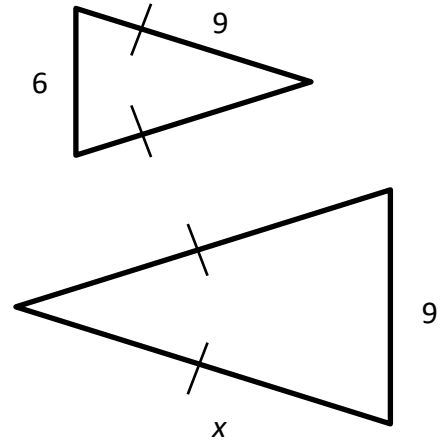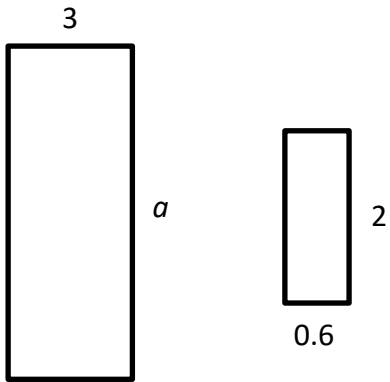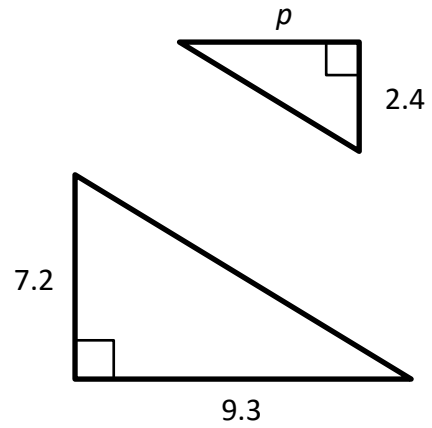
p = …………………

(GMMC3, 8 marks)

35 a) Calculate the length labelled x in these triangles. Give your answers to two decimal places if appropriate.
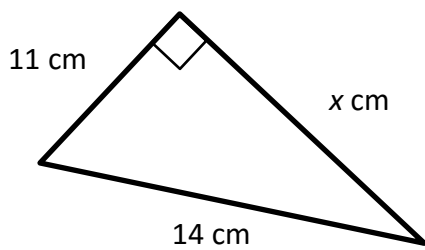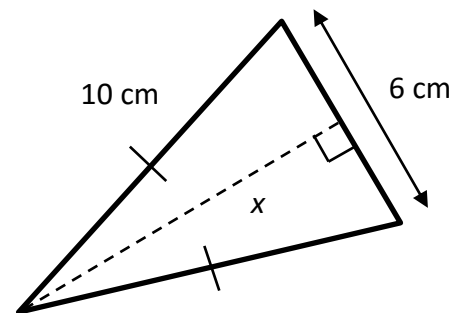
i)

12 cm

x cm

7 cm

x = ……………………

ii)

10 cm

x cm

8 cm

x = ……………………

iii)

11 cm

x cm

14 cm

a = ……………………

iv)

10 cm

6 cm

x
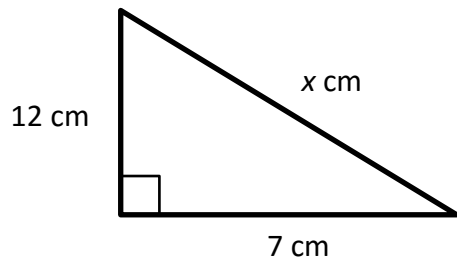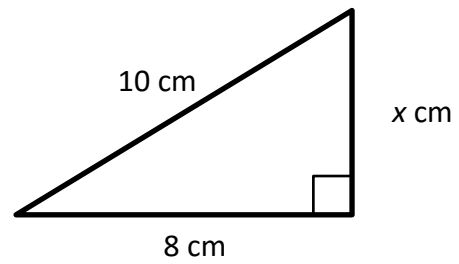
p = ……………………

b) Are these triangles right angled? Explain your answer.

	Side lengths	Right-angled?	Reason
i)	6 cm 7 cm 8 cm	Yes ☐ No ☐	
ii)	5 cm 12 cm 13 cm	Yes ☐ No ☐	
iii)	6 cm 9 cm 12 cm	Yes ☐ No ☐	

(GMMC4, 14 marks)

36   The table show the number of texts sent each day for two weeks.

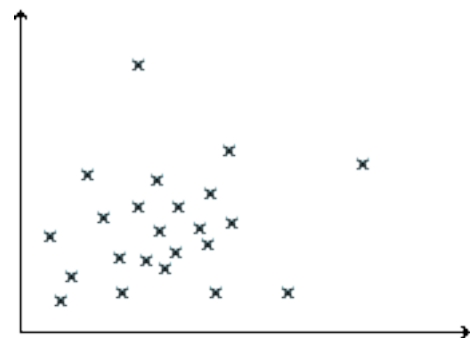| Week	1							2						
Day	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Number of texts	23	10	13	6	10	19	15	29	4	7	12	18	25	11

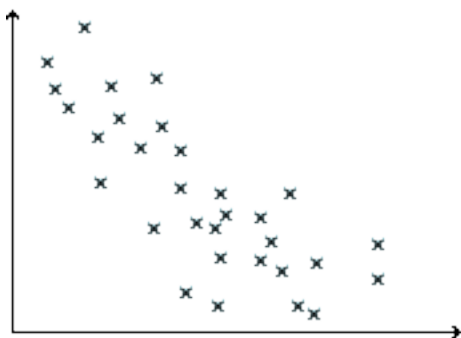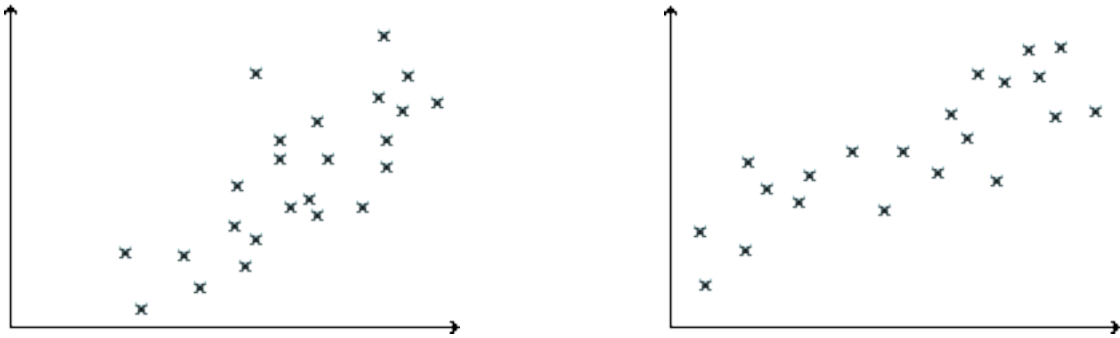Draw a time series graph to show this information.
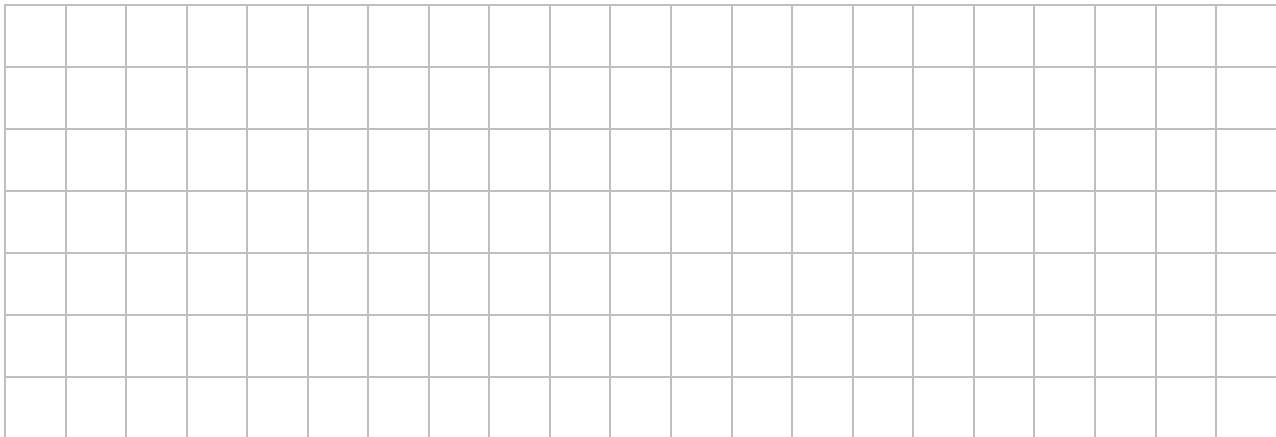
(S1, 3 marks)

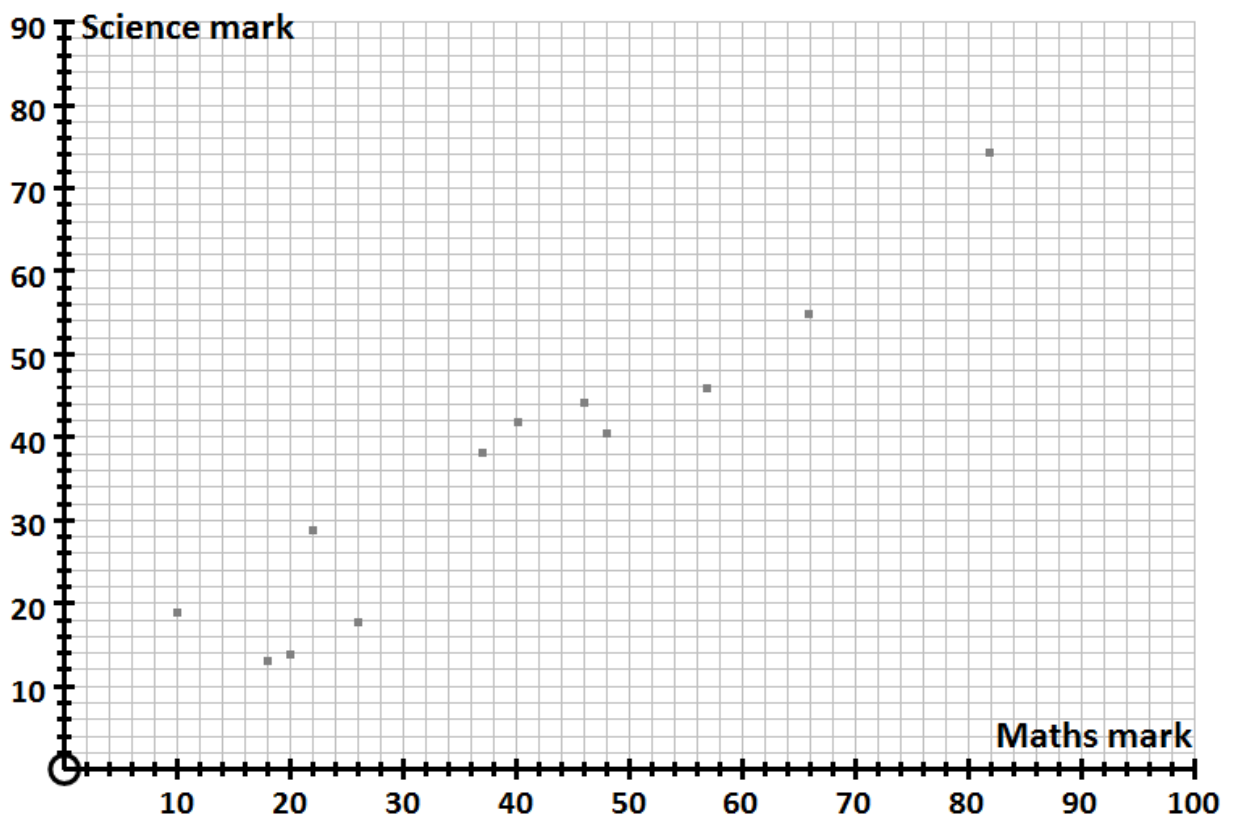**37 a)**   Here are some scatter graphs.

Draw a line of best fit on the scatter graph if possible.

If not possible, put a ring around the graph.

b) The scatter diagram shows the percentage scores of 12 pupils sitting maths and science exams.
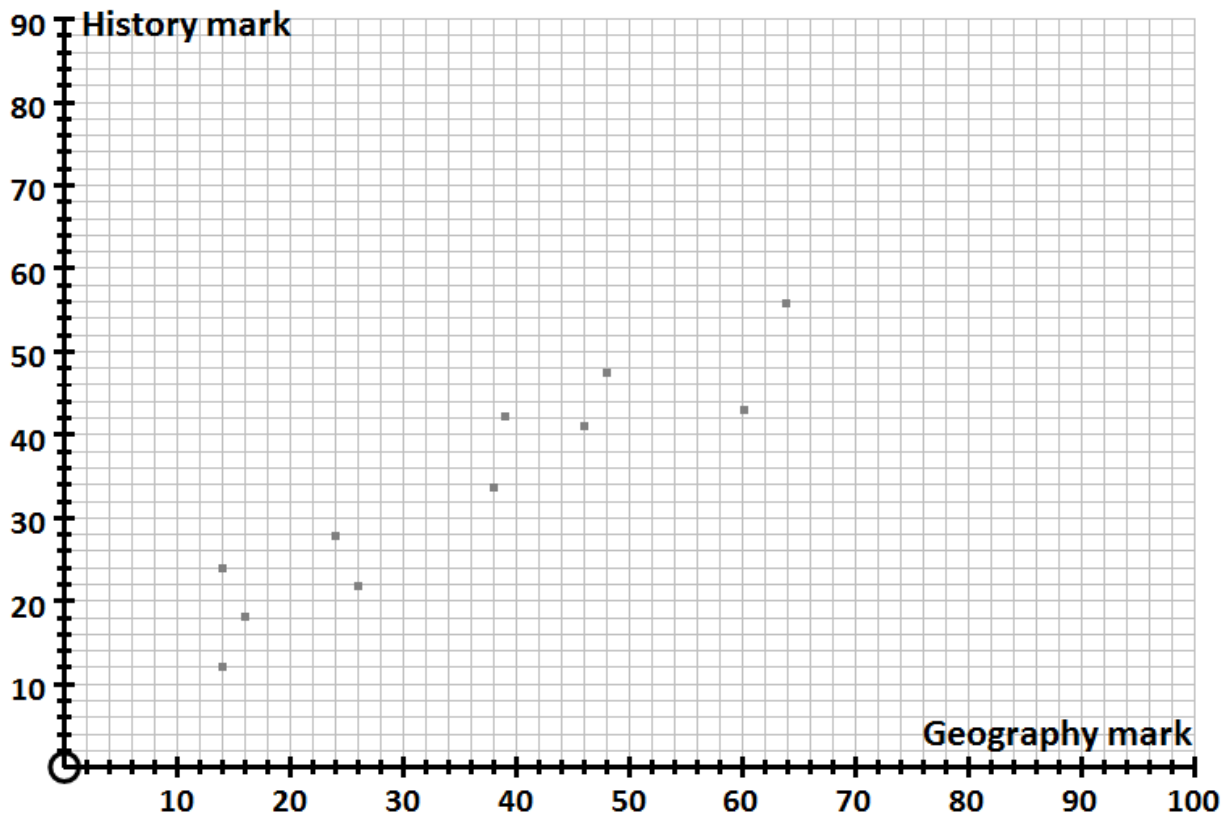

Tim scored 65 on the maths exam.

By drawing a line of best fit, predict the Tim's science percentage score.   ....

((S2, 6 marks)

38   The scatter diagram shows the percentage scores of 11 pupils sitting history and geography exams.

a)   Explain why the scatter diagram should not be used to predict scores beyond 80.

b)   Henry says that the scatter diagram means that if you get a high score in history you are bound to get a high score in geography.

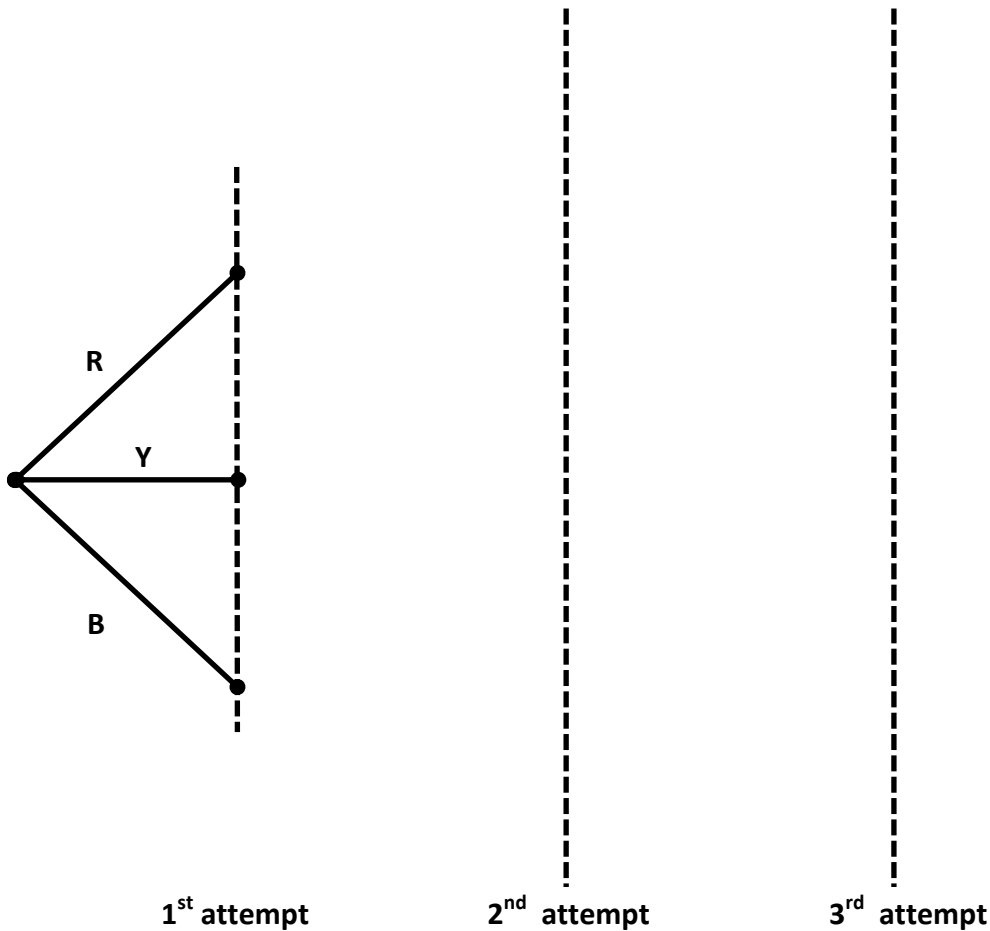Do you agree? Explain why.

(S3, 4 marks)

39   A spinner has three colours: red, yellow and blue.

The spinner is spun 3 times and the colour recorded each time.

By completing the tree diagram, enumerate the possible outcomes.

R

Y

B

1ˢᵗ attempt   **2ⁿᵈ attempt**   **3ʳᵈ attempt**

(P1, 2 marks)

40   Ruth rolls a fair six-sided die 100 times. Hazel rolls the same die 1000 times.

Helen predicts that:

- each number will appear 1000 ÷ 6 times for Hazel
- each number will appear 100 ÷ 6 times for Ruth

Do you agree with Helen's prediction? Explain your reasoning.

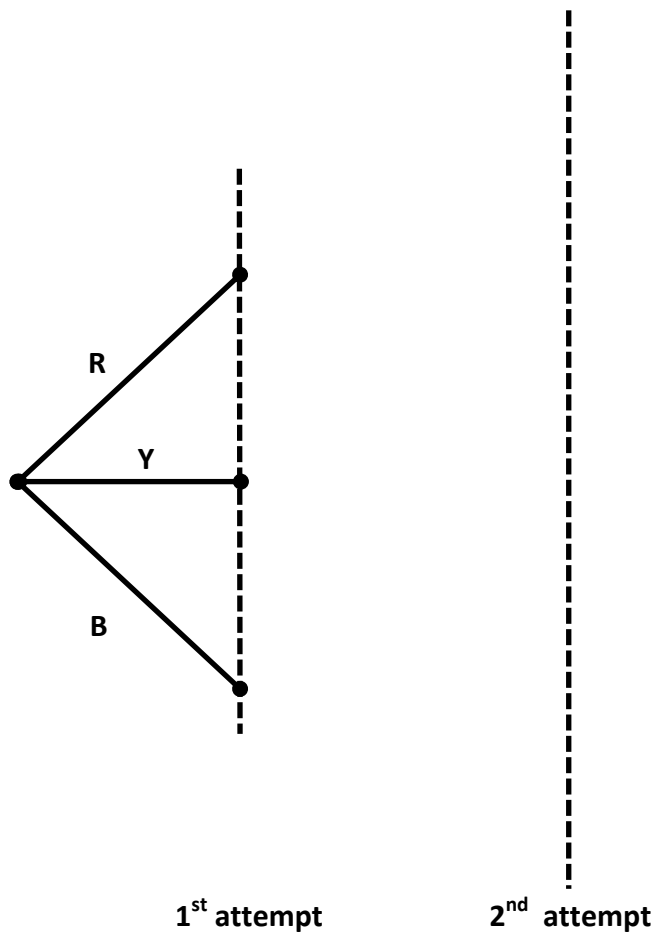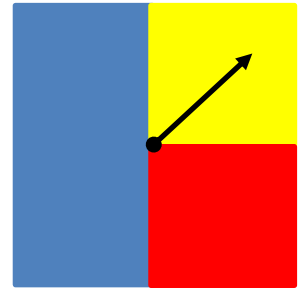*(P2, 2 marks)*

41 a)   A spinner has 3 colours: red, yellow and blue.

The spinner is spun twice and the colour recorded each time.

i)   Complete the tree diagram to show all possible outcomes

1ˢᵗ attempt   **2ⁿᵈ attempt**

ii)   Using the tree diagram, calculate the probability of getting blue followed by orange.

…………………

iii)   Using the tree diagram, calculate the probability of getting the same colour on both occasions.

…………………

b) A bag consists of six balls: 3 red, 2 black and one white ball.

A ball is taken out, the colour recorded and not replaced in the bag.

A second ball is taken out, the colour recorded and not replaced in the bag.

i) Construct a tree diagram to show all possible outcomes

ii) Use the tree diagram to calculate the probability of picking 2 black balls

…………………….

iii) Use the tree diagram to calculate the probability of picking 2 balls of different colours.

…………………….

(P3, 14 marks)

Structure and Calculation	Measures and Accuracy	Notation, Vocabulary and Manipulation	Graphs	Solving Equations and Inequalities	Sequences	Ratio, Proportion and Rates of Change	Properties and Construction	Mensuration and Calculation	Statistics	Probability
NUMBER		ALGEBRA					GEOMETRY & MEASURES			

NOT GOT IT YET?

Key topics I need to work on: